# HEAT EXCHANGE WITH HORIZONTAL CYLINDERS IN A CENTRIFUGAL-BUBBLING BED

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The authors present results of an analysis of experimental data on heat exchange on horizontal cylinders in rotating gas-liquid flow. The hydrodynamic parameters of a thin film in a centrifugal-bubbling bed have been determined based on the mathematical modeling of a film flow in the case of a high-velocity gas-liquid flow about it. Results of a generalization of the data on heat exchange are given, and the hydrodynamic stability criteria for the film flow on the cylinder surface are evaluated.

Heat transfer to horizontal cylinders installed in a vortex chamber in which a rotating gas-liquid (or centrifugal-bubbling) bed was produced was studied in [1]. The experiments revealed extremely high values of the heat-transfer coefficients. The heat-flux density on the surface of the cylinders was  $(150-250) \text{ kW/m}^2$ .

Figure 1 gives the dependences of the heat-transfer coefficients  $\alpha$  on the gas velocity referred to the area of the vortex generator and on the cylinders' diameter. Their analysis is of significant interest since in rotating gas-liquid beds a significant factor determining the structure of the gas-liquid bed is constituted by the centrifugal forces that occur in the bed.

According to the method of generalization of experimental data on heat exchange in foam beds, it is necessary to be able to calculate the friction on the wall (heat-exchange surface) from which the "dynamic velocity" of friction  $v^*$  is determined.

To calculate the heat-transfer coefficient on horizontal cylinders in a foam bed, one recommends the generalized dependence

$$St = 0.562 \text{ Re}^{-0.76} \text{ Ga}^{0.15} .$$
 (1)

For a centrifugal-bubbling apparatus, it is proposed [3] to calculate the friction of a rotating gas-liquid bed against the walls of the vortex chamber from the relation

$$\tau = C_f \frac{\rho_s w_s^2}{2} \,. \tag{2}$$

Here the rotational velocity of the gas-liquid bed  $w_s$  is calculated from the theoretical dependence [3]

$$\frac{w_{\rm s}}{V_0} = \sqrt{2A\,\sin\alpha} - \left(1 - \frac{\delta_{\rm s}}{R}\right)A,\tag{3}$$

where

\*) Deceased.

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Fig. 1. Heat-transfer coefficient  $\alpha$  vs. gas velocity w'': 1) D = 4 mm; 2) 3; 3) 6; 4) 1.5.

Fig. 2. Scheme for calculation of heat exchange in a liquid film.

$$A = \frac{s}{C_f K (1 - \varphi)} \frac{\rho_2}{\rho_1}; \quad K = 1 - s + \frac{2\delta_s}{h} \left( 1 - \frac{1}{2} \frac{\delta_s}{R} \right)$$

From the condition of agreement of the calculation by (3) with experimental data, the value  $C_f = 0.008$  is recommended in [3] for the coefficient of friction  $C_f$ .

One can assume that the use of expression (2) for determination of the friction stress on the wall in analysis of the data on heat exchange in a centrifugal-bubbling bed is quite justified.

From the basic propositions of the method of generalization of experimental data on heat exchange in foam beds given in [2] it follows that the main thermal resistance in heat exchange on the surface in gas-liquid flow is exhibited by a liquid film of thickness  $\delta$  that is formed on this surface. According to the assumption of [2], the heat-transfer coefficient on a cylinder in a foam bed is defined by the simple expression  $\alpha = jc_p$  which, in dimensionless form, appears as

$$Nu = Re_{f} Pr.$$
(4)

The hydrodynamic parameters of a liquid bed on the surface in the case of high-velocity flow of a gas or a gas-liquid mixture about this bed are determined by a number of hydrodynamic criteria of importance, among which are the following criteria [4]: the S. S. Kutateladze criterion of dynamic stability k, the Laplace criterion Lp, and the criterion of droplet breakdown Kd.

Critical conditions under which the processes of change in the hydrodynamic parameters of thin liquid beds occur are usually expressed in the form of the equality of the criteria k and Lp to certain numerical values. For example, from the data of [5] liquid films are sputtered when Lp = 4. From the data of [4] the sputtering of droplets is observed when Lp = 10.7. If account is taken of the effect of the liquid's viscosity on the sputtering of droplets or films, one obtains functional dependences of the form k = f(Kd) or Lp = f(Kd). It was of interest to evaluate the magnitudes of the criteria of hydrodynamic stability of the film flow as applied to the conditions of flow of a gas-liquid mixture about the heat-exchange surface in a vortex chamber.

The determination of the method of finding the parameters of the film flow on the cylinder's surface in the case of rotating gas-liquid flow about it is crucial for solution of the problems formulated. The most important method is an experimental method. However, in this case the method of mathematical modeling was applied as the first stage of investigation. It was assumed that the thickness of the film could be determined from the condition of equality of the calculated heat-transfer coefficient to its experimental value. The scheme of the considered problem is shown in Fig. 2.

In forced flow of the film, its hydrodynamics is described by the equation



Fig. 3. Generalization of experimental data on heat exchange on horizontal cylinders in a centrifugal-bubbling bed: 1) D = 4 mm; 2) 3; 3) 6; 4) 1.5; 5) calculation from formula (1).

$$\mu \frac{dw}{dy} = \tau = \text{const} .$$
<sup>(5)</sup>

Here, as has already been noted, the friction stress  $\tau$  can be calculated from formula (2), with it having a constant value across the thickness of the film. The velocity profile is determined from the solution of Eq. (5), after which the average velocity of the film flow is found. Finally, we obtain the relationship of the film thickness with the Reynolds number of the film flow in the form

$$\delta = \frac{v}{v} \sqrt{2Re_f} \,. \tag{6}$$

The heat exchange in the liquid film was calculated based on a numerical solution of the equation of convective heat exchange:

$$w \frac{\partial T}{\partial x} = a \frac{\partial^2 T}{\partial y^2}.$$
<sup>(7)</sup>

On the surface of the film and on the wall we set conditions of the first kind: for y = 0,  $T = T_w$  (the temperature of the wall of the experimental area); for  $y = \delta$ ,  $T = T_s$  (the temperature of the gas-liquid bed measured experimentally); for x = 0,  $T = T_0$ , this initial value being selected from the temperature range  $T_w < T_0 < T_s$ .

Equation (7) was approximated using a two-layer implicit six-point scheme on a rectangular grid consisting of integral and half-integral nodes in accordance with recommendations [6]. The advantage of calculating by an implicit scheme lies in the stability of the latter when the parameters of the problem change. The calculational algorithm was tested earlier in calculating the heat exchange on the initial portion of the film flow [7], where good agreement between the results of the numerical calculation and those of a calculation from analytical dependences, recommended in the literature [8], was shown.

After the calculation of the temperature profile we determined the local and length-average values of the heat-transfer coefficient on the wall and on the film surface. In calculating, the Reynolds number of the film was a free parameter. The magnitude of the assigned Reynolds number was used to calculate the value of the film thickness. Using the average heat-transfer coefficient obtained in the numerical calculation, we determined the heat flux q, which was compared with its experimental value. When they did not coincide, the Reynolds number was corrected. The calculation continued until the assigned accuracy of their agreement (~1%) was reached. The correctness of the numerical calculation was verified by comparing it with a calculation from the relation



Fig. 4. Nusselt number Nu vs. combination Re<sub>f</sub> Pr: 1) D = 4 mm; 2) 3; 3) 6; 4) 1.5; 5) calculation from formula (4).

Fig. 5. Laplace criterion Lp vs. breakdown parameter Kd: 1) D = 4 mm; 2) 3; 3) 6; 4) 1.5.

$$q = \lambda \frac{T_{\rm w} - T_{\rm s}}{\delta},\tag{8}$$

which holds true for the stabilization portion of the heat-transfer coefficient.

Figure 3 presents a comparison of the results of processing experimental data on the heat exchange on the cylinders in a centrifugal-bubbling bed in accordance with dependence (1). It is seen that the experimental data for cylinders of different diameters agree with each other satisfactorily in dimensionless coordinates. However, a certain disagreement with the calculation from dependence (1) occurs, which should be explained by the dissimilar procedures of calculation of the dynamic velocity of friction. The obtained set of experimental points can be generalized by a dependence of the form (1), in which the constant coefficient should be taken to be 0.381.

Since, in calculating, we obtained data on the magnitude of the film Reynolds number it became possible to directly verify relation (4).

Figure 4 presents the obtained data in the coordinates that follow from dependence (4). Here the diagonal line (points 5) represent the calculation from this dependence. As is seen, for small values of the Reynolds number a correction of the obtained data with the calculation from dependence (4) occurs. However, as the Reynolds number increases, we observe a deviation from this dependence, which is probably due to the presence of "reaching" of the self-similar regime by the heat-transfer coefficient when the film thickness stops being dependent on the velocity of the gas-liquid flow about it [9] in the case of heat exchange in gas-liquid beds.

As applied to the analysis of stability of the film flow as it interacts with the gas-liquid flow, the expression for the Laplace stability criterion is modified in the following manner:

$$Lp = \frac{\rho_s w_s^2 \delta}{\sigma} \approx \frac{\rho_1 \left(1 - \varphi\right) w_s^2 \delta}{\sigma}.$$
(9)

The breakdown parameter was also calculated from the rotational velocity of the bed:

$$\mathrm{Kd} = \frac{w_{\mathrm{s}}\,\mu}{\sigma}\,.\tag{10}$$

Figure 5 gives the dependences of the Laplace criterion on the breakdown parameter using the obtained data on the film thickness on the surface of cylinders of different diameters. It is seen that in the centrifugal-bubbling bed the Laplace stability criterion changes continuously with increase in the rotational velocity of the bed, i.e., each regime of rotation of a gas-liquid bed, from the viewpoint of the theory of stability, can be realized only with a certain combination of its stability criteria, and their change occurs continuously with change in the mean-flow-rate velocity of the gas and accordingly the rotational velocity of the bed. This circumstance fundamentally distinguishes rotating gas-liquid beds from those in the gravity field, where, as has been noted earlier, the stability criteria have quite definite critical values.

### CONCLUSIONS

1. We generalized experimental data on heat exchange on the surface in a centrifugal-bubbling bed based on the procedure developed in calculating the heat exchange in foam beds in the gravity field and modified as applied to a rotating gas-liquid bed.

2. Based on the values of the Reynolds numbers of a film flow, we showed a fundamental possibility of describing heat exchange on a cylinder in a gas-liquid flow by the simple relation (4). This indicates that the mechanism of heat removal in this case is determined by the heat capacity and the flow rate of the liquid phase that gets to the cylinder's surface. This conclusion agrees with the results of the investigations of the heat exchange on cylinders in gas flows with liquid droplets [10–14].

3. It was shown that a continuous change in stability criteria that characterize the minimum thickness of a liquid film on the surface in the case of gas-liquid flow about it occurs in a centrifugal-bubbling bed.

## NOTATION

w'', mean-flow-rate velocity of the gas in the centrifugal-bubbling apparatus (per total area of the vortex generator), m/sec;  $w_s$ , linear rotational velocity of the gas-liquid bed, m/sec;  $V_0$ , velocity of the gas in the vortex-generator holes, m/sec; sin  $\alpha$ , sine of the angle between the direction of channels in the vortex generator and the radius of the vortex chamber; R, radius of the vortex chambers, m;  $\delta_s$ , thickness of the rotating gas-liquid bed, m; s, "useful cross section" of the vortex generator;  $C_f$ , coefficient of friction;  $\tau$ , friction stress, N/m<sup>2</sup>;  $v^* = \sqrt{\tau/\rho_1}$ , dynamic velocity of friction on the wall, m/sec;  $\delta$ , film thickness, m;  $\phi$ , gas content of the bed;  $\rho_1$  and  $\rho_2$ , density of the liquid and the gas, respectively, kg/m<sup>2</sup>; h, height of the vortex generator, m;  $\rho_s \approx \rho_1(1 - \phi)$ , density of the gas-liquid mixture, kg/m<sup>3</sup>;  $c_p$ , heat capacity of the liquid, J/(kg·K); v, kinematic viscosity of the liquid,  $m^2/sec$ ; D, outside diameter of the cylinder, m; j, density of the liquid flow to the cylinder surface, kg/(m<sup>2</sup>·sec);  $\sigma$ , coefficient of surface tension, N/m;  $\mu$ , coefficient of dynamic viscosity of the liquid, Pa·sec;  $\lambda$ , thermal conductivity of the liquid, W/(m·K);  $\alpha$ , heat-transfer coefficient,  $kW/(m^2 \cdot K)$ ; T, temperature of the liquid, K; q, heat-flux density,  $W/m^2$ ; w, local velocity of the liquid in the film, m/sec; a, thermal diffusivity of the liquid,  $m^2/sec$ ; x, y, coordinates in the direction of discharge of the film and normal to its surface, respectively, m; St =  $\alpha/\rho_1 c_p v^*$ , Stanton number; Re =  $v^* D/v$ , Reynolds number; Ga =  $gD^3/v^2$ , Galilean criterion; Nu =  $\alpha D/\lambda$ , Nusselt number constructed from the cylinder diameter;  $\operatorname{Re}_{f} = \overline{w}\delta/\nu = jD/(\rho_{1}\nu)$ , Reynolds number of the film;  $\overline{w}$ , average value of the liquid velocity in the film, m/sec;  $Pr = \rho_1 c_p v / \lambda$ , Prandtl number;  $k = w \sqrt[n]{\sqrt{\rho_2}} (g\sigma(\rho_1 - \rho_2))^{1/4}$ , Kutateladze stability criterion;  $Lp = v \sqrt[n]{\rho_2} (g\sigma(\rho_1 - \rho_2))^{1/4}$  $\rho_2(w'')^2 \delta/\sigma$ , Laplace criterion; Kd =  $w'' \mu/\sigma$ , breakdown criterion. Subscripts and superscripts: s, rotating gas-liquid bed; f, liquid film; 1, liquid; 2, gas; 0, initial values of parameters.

#### REFRENCES

- 1. A. R. Dorokhov, A. A. Kirsanov, and V. I. Kazakov, Sib. Fiz.-Tekh. Zh., Issue 6, 3-7 (1992).
- 2. A. P. Burdukov, A. R. Dorokhov, A. V. Gorin, and O. Yu. Kileeva, *Teplofiz. Aeromekh.*, 4, No. 3, 319–323 (1997).
- 3. M. I. Shilyaev and A. R. Dorokhov, Teplofiz. Aéromekh., 5, No. 2, 189-194 (1998).

- 4. S. S. Kutateladze and M. A. Styrikovich, *Hydrodynamics of Gas-Liquid Systems* [in Russian], Moscow (1976).
- 5. V. N. Bykov and M. E. Lavrent'ev, Inzh.-Fiz. Zh., 31, No. 8, 782-787 (1976).
- 6. V. M. Paskonov, V. I. Dolezhaev, and L. A. Chudov, *Numerical Modeling of Processes of Heat and Mass Transfer* [in Russian], Moscow (1984).
- 7. M. I. Shilyaev, A. R. Dorokhov, and O. Yu. Kileeva, *Izv. Vyssh. Uchebn. Zaved., Stroitel'stvo*, Nos. 11–12, 130–134 (1998).
- 8. V. E. Nakoryakov and A. V. Gorin, *Heat and Mass Transfer in Two-Phase Systems* [in Russian], Novosibirsk (1994).
- 9. A. R. Dorokhov, M. I. Shilyaev, and V. I. Kazakov, Sib. Fiz.-Tekh. Zh., Issue 3, 31-36 (1991).
- 10. I. T. El'perin, Inzh.-Fiz. Zh., 4, No. 8, 30–35 (1961).
- 11. Goldstin, Yan Dsi-Ven, and Clark, Teploperedacha, No. 2, 80-90 (1967).
- 12. Hodgson, Seiterbeck, and Sunderland, Teploperedacha, No. 4, 96–103 (1968).
- 13. I. C. Finlay, Can. J. Chem. Eng., 49, 333–339 (1971).
- 14. A. N. Khoze and V. A. Shchennikov, Teor. Osn. Khim. Tekhnol., 8, No. 3, 460-463 (1974).